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L. were published. In one of these letters it was affirmed that Mr. Lenhart had done more for the Diophantine Analysis than any other man that had ever lived. In another, Professor G., after expressing a desire to publish Lenhart's tables relating to squares and cubes, remarked that the printing would be attended with no inconsiderable expense. Mr. L., who was an inveterate smoker, at once, as he informed me, gave up his cigars, and appropriated the money thus saved to meet the expense of publication.

Of the distressing accident by which Mr. Lenhart, when a young man, was maimed for life, of the consequent blighting of his temporal prospects, and the intensity of his physical sufferings during a period of 28 years, it is unnecessary to speak, as the circumstances are detailed at some length in the article to which I have already referred. He died at Frederick, Md., on the 10th of July, 1840.

ON SUPPLYING OMISSIONS IN A CLOSED SURVEY.

BY CLEVELAND ABBE, WASHINGTON, D. C.

The problem of supplying any omissions in a closed survey which is treated by Prof. Philbrick in the ANALYST, p. 116, is one that I had occasion to explain more fully than is done in Gillespie's Treatise, while teaching the engineering students, in 1859, at Ann Arbor. A portion of my analytical solution of the problem was published, in 1861, in the Journal of the Franklin Institute, by Prof. DeVolson Wood with valuable additions by himself; and as it may be interesting to surveyors to look at the subject from this as well as the geometrical point of view, I will add the following to what Professors Wood and Philbrick have written.

Let the ordinary bearings (x) of the sides be converted into azimuthal angles by the following rules:

N. x° *E.* becomes azimuth x°; *S.* x° *E.* becomes azimuth $180^\circ - x^\circ$;
N. x° *W.* " " $360^\circ - x^\circ$; *S.* x° *W.* " " $180^\circ + x^\circ$;

the converse of which need not be repeated here.

Let the known sides be a_1, a_2, &c.; the known azimuths, b_1, b_2, &c.;
" unknown " " x_1, x_2, &c.; " unknown " y_1, y_2, &c.

The sides being always considered as positive numbers, the general equations of condition in a perfect survey are

$$\Sigma \text{ Dep.} = a_1 \sin b_1 + a_2 \sin b_2 + a_3 \sin b_3 \text{ \&c.} = 0,$$

$$\Sigma \text{ Lat.} = a_1 \cos b_1 + a_2 \cos b_2 + a_3 \cos b_3 \text{ \&c.} = 0.$$

From which any two quantities, if unknown, may be found by elimination.

The following are the six possible cases of omissions.

I. One distance is omitted. We find the required side from

$$x_1 = \frac{D}{\sin b_1} \qquad \text{or } x_1 = \frac{L}{\cos b_1},$$

where D and L indicate respectively the sums of the known Departures or Latitudes.

II. One bearing is omitted. The required azimuth is found from

$$\sin y_1 = \frac{D}{a_1} \qquad \text{or } \cos y_1 = \frac{L}{a_1}, \qquad \text{or } \tan y_1 = \frac{D}{L}$$

In both these cases we have two equations for determining one unknown and it is best to compute them both. If the two resulting values are identical the numerical processes are correct.

III. The length and bearing of one side are omitted. The general equations give

$$\left.\begin{aligned} x_1 \sin y_1 &= D, \\ x_1 \cos y_1 &= L, \end{aligned}\right\} \text{whence } \tan y_1 = \frac{D}{L} \text{ and } x_1 = \frac{D}{\sin y_1} = \frac{L}{\cos y_1}.$$

IV. The length of one side and bearing of another are omitted. The general equations become

$$x_1 \sin b_1 + a_2 \sin y_2 = D,$$
$$x_1 \cos b_1 + a_2 \cos y_2 = L.$$

Whence $$\sin (y_2 - b_1) = \frac{1}{a_2}\Big(D \cos b_1 - L \sin b_1\Big).$$

This equation gives two values of $y_2 - b_1$ i. e. θ and $180° - \theta$, (whence result two values of y_2, i. e. $\theta + b_1$ and $180° + b_1 - \theta$, and therefore two values of x_1), so long as we have

$$D \cos b_1 - L \cos b_1 < a_2.$$

There will be but one solution when this inequality disappears, and there will be no possibility of closing the survey (indicating some error in the work of the surveyor or computer) if $D \cos b_1 - L \cos b_1 > a_2$.

V. The lengths of any two sides are omitted. We have in this case

$$x_1 \sin b_1 + x_2 \sin b_2 = D,$$
$$x_1 \cos b_1 + x_2 \cos b_2 = L;$$

whence we find x_1 and x_2 without ambiguity. One or the other of these lines may however result negative, and therefore the corresponding side of the survey will be imaginary.

VI. The bearings of any two sides are omitted. The general equations become

$$a_1 \sin y_1 + a_2 \sin y_2 = D,$$
$$a_1 \cos y_1 + a_2 \cos y_2 = L.$$

Introducing the auxilliary quantities m and θ such that

$$D = m \cos \theta \qquad \text{and } L = m \sin \theta$$

we have

$$\sin(\theta + y_2) = \frac{m^2 + (a_2^2 - a_1^2)}{2ma_2},$$

$$\sin(\theta + y_1) = \frac{m^2 - (a_2^2 - a_1^2)}{2ma_1};$$

whence are found two values for y_2 and y_1 respectively: these values of course become imaginary when the sine of $\theta + y_2$ or y_1 exceeds unity.

SOLUTION OF PROBLEM ON PAGE 40, *NUMBER* 2.

BY PROF. WILLIAM WOOLSEY JOHNSON, ANNAPOLIS, MD.

Two parabolas may be passed through four given points. The *directions* of their axes may be found by the following method given in Newton's Universal Arithmetic. Call the points A, B, C and D, join AB, BC, and CD; through A draw a parallel to CD, and through D a parallel to BC meeting the last parallel in M, and meeting AB in N. Construct a mean proportional to DM and DN, and lay it off in both directions from D on the line DM, the lines joining A with the points so found will be parallel to the axes of the two parabolas which may be passed through A, B, C, and D. [These directions may also be found by producing AB and CD to meet in O, laying off from O on OA a mean proportional to AO and OB, and on OD in both directions a mean proportional to OD and OC, the lines joining the points so found will have the required directions.]

Having selected the direction of the axis, the solution may be completed as follows; produce BC to meet the diameter already drawn through A, produce AB to meet a diameter through D; join these points of intersection, and produce CD to meet the joining line, the line joining this last intersection with A will be a tangent to the required parabola. [This is an application of Pascal's Theorem, the hexagon being reduced to three chords, two diameters, or chords meeting the curve at infinity, and a tangent.] A tangent at one of the other points, say D, may now be drawn by bisecting the chord AD with a diameter, since the tangents at D and A will meet this diameter in the same point. Having now the tangents at A and D, lines making at A and D the same angles with these tangents which the latter make with the diameters, will meet in the focus. Hence the axis may be found; and, bisecting a subtangent, the vertex.